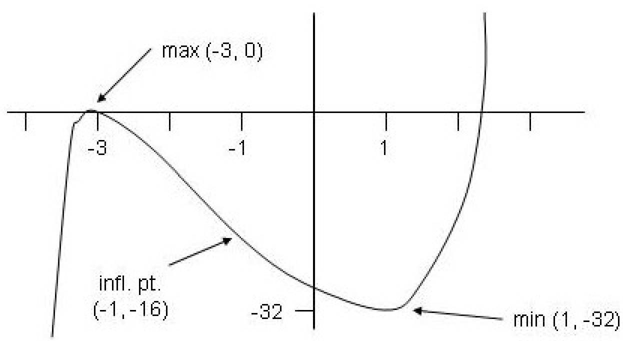
Math 141 **Test #3 Solutions** **Fall 2004**

1. (a) Find $f'(x) = 3x^2 + 6x - 9$. Set it equal to 0 and solve for x. Get $x = -3$ and $x = 1$. Choose test points, say $x = -4, 0$, and 2. Compute that $f'(-4) > 0$, $f'(0) < 0$, and $f'(2) > 0$. So f is increasing on $(-\infty, -3)$ and $(1, \infty)$ and f is decreasing on $(-3, 1)$.
   (b) Based on part (a), a relative max of f occurs at $x = -3$ and a relative min occurs at $x = 1$. The relative max is $f(-3) = 0$ and the relative min is $f(1) = -32$.
   (c) Find $f''(x) = 6x+6$. Set it equal to 0 and solve for x. Get $x = -1$. Choose test points, say $x = -2$ and 0. Compute that $f''(-2) < 0$ and $f''(0) > 0$. So f is concave down on $(-\infty, -1)$ and concave up on $(-1, \infty)$.
   (d) Based on part (c), an inflection point of f occurs at $x = -1$. The inflection point is $(-1, -16)$.

2. Your sketch should look something like this:



1.jpg

3. Since the numerator and denominator have the same degree, the limit is the ratio of their leading coefficients, namely 5/2.

4. Find $f'(x) = 3 - 10000/x^2$. Set it equal to 0 and solve for x. Get $x = 100/\sqrt{3} = 57.735$. Use the first derivative test to confirm this. Use test points, say $x = 50$ and 100. Compute that $f'(50) = 3 - 4 < 0$ and $f'(100) = 3 - 1 > 0$. Thus, a min occurs at $x = 57.735$. That is the width. The length is $5000/57.735 = 86.6$, since the area must be 5000.

5. The function to use is $f(x) = x^3 - 10$. So $f'(x) = 3x^2$. Then the formula is
$$x_{n+1} = x_n - \frac{x_n^3 - 10}{3x_n^2}.$$
If $x_1 = 2$, then $x_2 = 2.166667$ and $x_3 = 2.147633745$.

6. According to the Pythagorean theorem, $r^2 + (\frac{h}{2})^2 = R^2$. So $r = \sqrt{R^2 - \frac{1}{4}h^2}$. Substitute this into the formula for volume:
$$V = \pi r^2 h = \pi(R^2 - \frac{1}{4}h^2)h = \pi R^2 h - \frac{\pi}{4}h^3.$$

Find the derivative: $V' = \pi R^2 - \frac{3\pi}{4}h^2$. Set this equal to 0 and solve for h. Get $h = \frac{2R}{\sqrt{3}}$. For the second derivative test, find $V'' = -\frac{3\pi}{2}h$. Evaluate this at $h = \frac{2R}{\sqrt{3}}$ and get a negative result, indicating a max.

7. In the equation $h = b\tan\theta$, let $b = 100$. Then differentiate the equation to get

$$dh = 100\sec^2\theta d\theta.$$

Let $d\theta$ be $1°$, expressed in radians ($1° = \frac{\pi}{180}$ radians). Let θ be $30°$. Substitute and compute the value of dh. Get $dh = 2.327$.